Background Suppression for $\mu \to e\gamma$ with Polarized Muons

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A search for the lepton-flavor violating $\mu^+ \to e^+\gamma$ decay using polarized muons is revisited in terms of suppression of the serious background arising from an accidental coincidence between a e^+ in the normal muon decay and a high energy photon in the radiative muon decay, $\mu^+ \to e^+\nu\overline{\nu}\gamma$. It is found that a high energy photon in $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay is preferentially emitted parallel to the muon spin direction, similarly to e^+ in the normal muon decay. The selective measurements of either e^+s or photons moving antiparallel to the muon spin direction would suppress the accidental background for $\mu^+ \to e^+\gamma$ with right-handed and left-handed e^+s respectively.

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A search for lepton-flavor violating (LFV) processes such as $\mu^+ \to e^+\gamma$ decay and $\mu^- - e^-$ conversion in a nucleus has been attracting much theoretical and experimental interest, since various theoretical models with physics beyond the Standard Model predict a large LFV effect. In particular, supersymmetric grand unification (SUSY-GUT) predicts large branching ratios for those LFV processes, which are only one or two orders of magnitude lower than the current experimental limits [1]. Some other theoretical extensions to the Standard Model also predict a large branching ratio [2,3].

The event signature of $\mu^+ \to e^+\gamma$ is that a e^+ and a photon are in coincidence, and moving colinear back-to-back with their energies equal to a half of the muon mass ($m_\mu/2 = 52.8$ MeV). The current experimental upper limit is 4.9×10^{-11} at 90 % confidence level [4].

One of the major background to the search for $\mu^+ \to e^+\gamma$ is a radiative muon decay $\mu^+ \to e^+\nu\overline{\nu}\gamma$ (branching ratio is 1.4 % for $E_\gamma > 10$ MeV). When e^+ and photon are

emitted back-to-back with two neutrinos carrying off little energy, it becomes a serious physics background to $\mu^+ \to e^+\gamma$. The other background, which turns out more important in a new generation experiment with a very high rate of stopped muons, is an accidental coincidence of a e^+ in a normal muon decay, $\mu^+ \to e^+\nu\overline{\nu}$, accompanied by a high energy photon. The sources of a high energy photon might be either that in $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay, or external bremsstrahlung or annihilation-in-flight of e^+s in the normal muon decay.

In our previous paper by two of the authors [5], the importance has been emphasized of the use of polarized muons in a search for $\mu^+ \to e^+\gamma$. With the use of polarized muons, a right-handed e^+ (e^+_R) in $\mu^+ \to e^+_R\gamma$ decay follows a $1 - P_\mu \cos\theta_e$ distribution, whereas a left-handed e^+ (e^+_L) in $\mu^+ \to e^+_L\gamma$ does a $1 + P_\mu \cos\theta_e$ distribution, where θ_e is an angle of the e^+ emission with respect to the muon polarization (P_μ). This angular distribution is useful to discriminate different models. More importantly, it is shown that the physics background from the radiative muon decay has a e^+ following approximately a $1 + P_\mu \cos\theta_e$ distribution when the energy resolution of photon detection is worse than that of e^+. Also for the accidental background, e^+ in the normal muon decay is known to follow a $1 + P_\mu \cos\theta_e$ distribution. They imply that the selective measurement of e^+s antiparallel to the muon polarization direction would suppress these backgrounds, improving a sensitivity of the search only for $\mu^+ \to e^+_R\gamma$. No background suppression, however, is expected for $\mu^+ \to e^+_L\gamma$.

In this Letter, we present our further studies on the accidental backgrounds with polarized muons. In particular, in order to investigate the accidental background for $\mu^+ \to e^+_L\gamma$ decay, we examine the angular distribution of a high energy photon from $\mu^+ \to e^+\nu\overline{\nu}\gamma$. It is found to be emitted preferentially along the muon spin direction; namely follows a $1 + P_\mu \cos\theta_\gamma$ distribution, where θ_γ is an angle of the photon direction with respect to the muon spin direction. It implies that the accidental background could be suppressed for $\mu^+ \to e^+_L\gamma$ where high energy photons must be detected at the opposite direction to the muon polarization. This is the same suppression mechanism seen for $\mu^+ \to e^+_R\gamma$ where high energy e^+s going antiparallel to the muon polarization direction are measured. As a result,

the selective measurements of either e^+s or photons antiparallel to the muon spin direction would give the same accidental background rejection for $\mu^+ \to e^+_R\gamma$ and $\mu^+ \to e^+_L\gamma$ decays respectively.

To examine the angular distribution of a high energy photon in $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay, we calculated the differential decay width of the radiative muon decay, retaining the positron mass (m_e), as a function of e^+ energy (E_e) and photon energy (E_γ) [6]. Those energies are normalized to a half of the muon mass ($m_\mu/2$), namely $x = 2E_e/m_\mu$ and $y = 2E_\gamma/m_\mu$. From the four-body kinematics, the ranges of x and y are given as follows; y varies from 0 to $1-r$ where $r = (m_e/m_\mu)^2$. For $0 < y \leq 1-\sqrt{r}$, x changes from $2\sqrt{r}$ (*i.e.* the normalized positron rest mass) to $1+r$, and for $1-\sqrt{r} < y \leq 1-r$, $(1-y)+r/(1-y) \leq x \leq 1+r$. The differential decay width thus obtained is identical to that in Reference [7] when the positron mass is ignored. Since the e^+s in the radiative muon decay do not contribute to the accidental background, the differential decay width should be integrated over the e^+ energy and angle between e^+ and photon ($\theta_{e\gamma}$) in the kinematically allowed region. Only high energy photons in the extreme kinematic case of $y \approx 1$ can slip into the signal region. Taking these into account, in the limit of $y \approx 1$, the differential branching ratio is approximately given by

$$dB(\mu^+ \to e^+\nu\overline{\nu}\gamma) = \Big[J(y)(1 + P_\mu \cos\theta_\gamma) + O((1-y)^2, \sqrt{r})\Big] dyd(\cos\theta_\gamma) \tag{1}$$

where $J(y)$ is given by

$$J(y) = \frac{\alpha}{2\pi}(1-y)\Big[\ln\frac{(1-y)}{r} - \frac{17}{6}\Big] \tag{2}$$

As seen in Eq.(1), the angular distribution of high energy photon ($y \approx 1$) follows a $1 + P_\mu \cos\theta_\gamma$ distribution when the higher order terms of $(1-y)$ are neglected. Equation (2) is derived under the assumption of $2\sqrt{r} \ll 1-y \ll 1$. It should be noted that Eq.(1) is consistent with the other calculations for the case of unpolarized muons [8]. A rate of the observed events in a real experiment can be estimated by integrating the spectrum over the photon energy resolution of the detector, or more precisely the width of the signal region. Taking δy to be a half width of the signal region for photon energy, the partial branching

ratio (denoted by b_γ) integrated over the signal region ($1-\delta y \le y \le 1-r$) can be calculated from Eq.(1) by

$$\begin{aligned} b_\gamma &= \int_{1-\delta y}^{1-r} dy \frac{dB(\mu^+ \to e^+\nu\overline{\nu}\gamma)}{dy} \\ &\approx \Big(\frac{\alpha}{4\pi}\Big)(\delta y)^2 \Big[\ln(\delta y) + 7.33\Big](1 + P_\mu \cos\theta_\gamma)d(\cos\theta_\gamma) \end{aligned} \quad (3)$$

From Eq.(3), it is shown that b_γ is roughly proportional to $(\delta y)^2$.

The numerical calculation of b_γ was also carried out based on the exact expression of the differential decay width. The effect of the positron mass was taken into account in the evaluation of kinematic boundaries of x and $\theta_{e\gamma}$, as well as in the expression of the differential decay width, although the latter turned out to be small, about a few %. In Fig.1 the numerically calculated b_γ is shown as a function of $\cos\theta_\gamma$ for various values of δy, 0.05, 0.03, and 0.01. Fig.1 also shows b_γ given in Eq.(3), which is in agreement with the numerically calculated b_γ. As seen in Fig. 1, the higher the energy of detected photon is (*i.e.* δy becomes smaller), the closer to the $1 + P_\mu \cos\theta_\gamma$ the angular distribution of b_γ becomes. For $\delta y = 0.03$ with $P_\mu = 100$ %, the constant term remaining at $\cos\theta_\gamma = -1$ is about 2.1 % of the value at $\cos\theta_\gamma = +1$.

The other sources of high energy photons ($y \approx 1$) are external bremsstrahlung and annihilation-in-flight of e^+s in the normal muon decay. Since e^+s in the normal muon decay is known to follow a $1 + P_\mu \cos\theta_e$ distribution, photons from these sources are also emitted preferentially along the muon spin direction. It is noted that they could be minimized experimentally by reducing materials around the target, and are found in previous experiments to be less significant than that from the radiative decay [9].

From these, it is concluded that both of the two major sources of accidental background, high energy photons ($y \approx 1$) from the radiative muon decay and high energy e^+ ($x \approx 1$) from the normal muon decay, follow $1+P_\mu \cos\theta$ distribution, where θ is either θ_e or θ_γ. In a search for $\mu^+ \to e^+_R\gamma$ where a e^+ is emitted antiparallel to the muon spin direction, the suppression of accidental background results from the selective measurement of e^+ moving antiparallel to the muon spin. In a search for $\mu^+ \to e^+_L\gamma$ where a high energy photon is antiparallel to

the muon spin direction, the same is the case if a e^+ is replaced with a photon. As a result, the accidental background could be suppressed for both $\mu^+ \to e^+_R\gamma$ and $\mu^+ \to e^+_L\gamma$ decays by the use of polarized muons.

A quantitative discussion of the suppression factor is given below. The rate of the accidental background (B_{acc}) normalized to the total decay rate can be estimated by

$$B_{acc} = R_\mu \cdot f_e^0 \cdot f_\gamma^0 \cdot (\Delta t) \cdot (\frac{\Delta\omega}{4\pi}) \cdot \eta \tag{4}$$

where R_μ is an instantaneous muon intensity. f_e^0 and f_γ^0 are defined in $f_e \equiv f_e^0(1 + P_\mu \cos\theta_e)(d\Omega_e/4\pi)$ and $f_\gamma \equiv f_\gamma^0(1 + P_\mu \cos\theta_\gamma)(d\Omega_\gamma/4\pi)$, where f_e and f_γ are an integrated fraction of the spectrum of e^+ in the normal muon decay and that of photon in $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay within the signal region, respectively. f_e^0 and f_γ^0 include their corresponding branching ratios. Here, for simplicity, the source of a high energy photon is assumed to come mostly from the radiative muon decay. Δt and $\Delta\omega$ are respectively a full width of the signal regions for timing coincidence and angular constraint of the back-to-back kinematics. η is a suppression factor of the accidental background. When η is small, the accidental background is suppressed. $\eta = 1$ for the case of unpolarized muons.

Given the sizes of the signal region, B_{acc} can be estimated. Take δx, δy, $\delta\theta_{e\gamma}$, δt to be a half width of the signal region for e^+ and photon energies, angle $\theta_{e\gamma}$ and relative timing between e^+ and photon, respectively. f_e^0 can be estimated by integrating the Michel spectrum of the normal muon decay from $1 - \delta x \leq x \leq 1 + r$, yielding $f_e^0 \approx 2(\delta x)$. f_γ^0 can be given from Eq.(3). $\Delta\omega/4\pi$ is $(\delta\theta_{e\gamma})^2/4$. For instance, to obtain quantitative estimation, take some reference numbers such as $\delta x = 0.005$, $\delta y = 0.03$, $\delta\theta_{e\gamma} = 0.01$ radian, $\delta t = 0.5$ nsec, and $R_\mu = 3 \times 10^8$ μ^+/sec, B_{acc} is 3×10^{-13}. Unless there are significant improvements made on the detector resolution, the accidental background might appear at a level of 10^{-13}.

With the use of polarized muons, however, the accidental background can be suppressed further. The selective measurement of either e^+s or photons antiparallel to the muon spin direction with a medium solid angle would provide a large background suppression factor. By taking account of the angular distributions of e^+s and photons which are supposed to be

back-to-back for the $\mu^+ \to e^+\gamma$ signal, the suppression factor η is given for polarized muons by

$$\eta \equiv \int_{\cos\theta_D}^{1} d(\cos\theta)(1+P_\mu\cos\theta)(1-P_\mu\cos\theta)/\int_{\cos\theta_D}^{1} d(cos\theta)$$
$$= (1-P_\mu^2) + \frac{1}{3}P_\mu^2(1-\cos\theta_D)(2+\cos\theta_D) \quad (5)$$

where θ_D is a half opening angle of detection with respect to the muon polarization direction. In Fig.2, the suppression factor of the accidental background is shown as a function of θ_D. To obtain a high suppression factor, the detector solid angle has to be adequately modest. For instance, for $\theta_D = 300$ mrad, an accidental background can be suppressed down to about 1/20 when P_μ is 100 %.

As seen in Eq.(5), however, if P_μ is not sufficiently high, η might be dominated by $1-P_\mu^2$, and beyond it no further suppression can be obtained even if making θ_D smaller. Therefore, it is inevitable to have P_μ as high as possible. The previous experiment [4] used a surface muon beam, which is muons from a decay of pions stopped near the surface of the pion production target. From their production mechanism, they are supposed to be 100 % polarized opposite to the muon momentum direction. In reality, the beam channel acceptance and multiple scattering in the production target will decrease the muon polarization, but it should be arranged to be at least 97 % or more. For instance, a muon polarization of 97 % will give us a suppression of about $\eta = 1/10$ for $\theta_D = 300$ mrad, as seen in Fig.2. Similarly, a deviation from a pure $1+\cos\theta$ distribution of e^+s and photons would introduce the same effects. For instance, for $\delta y = 0.03$, the residual at $\cos\theta_\gamma = -1$ changes a suppression factor from 1/20 to 1/8 for $\mu^+ \to e^+_L\gamma$ with $\theta_D = 300$ mrad and $P_\mu = 100$ %. This would be improved if δy becomes smaller. Since the e^+ energy resolution is better than the photon energy resolution, this effect is much smaller for $\mu^+ \to e^+_R\gamma$.

An opening angle of the detector of $\theta_D = 300$ mrad, taken as an example in this paper, gives a geometrical solid-angle coverage of 2.3 % for each side. If the $\mu^+ \to e^+\gamma$ signals are preferentially emitted, following a $1 \pm P_\mu\cos\theta$ distribution, towards the detector coverage, the effective signal acceptance could be increased further by a factor of two, resulting in the

additional improvement of the signal-to-background ratio by a factor of two. This solid angle would be large enough to yield a single event sensitivity to $\mu^+ \to e^+\gamma$ at the level of 10^{-14} or less for the muon beam intensity presently available, together with sufficient accidental background suppression thus discussed. It should be noted that the physics background from $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay is also estimated to be less than 10^{-14} for the detector resolutions given in this paper [5].

In conclusion, with polarized muons, a search for both $\mu^+ \to e^+_L\gamma$ as well as $\mu^+ \to e^+_R\gamma$ has the potential for improved sensitivity in terms of accidental background suppression. The suppression in the search for $\mu^+ \to e^+_R\gamma$ decay comes from the angular distribution of e^+s in the normal muon decay, whereas that for $\mu^+ \to e^+_L\gamma$ decay is due to the distribution of high energy photon in $\mu^+ \to e^+\nu\overline{\nu}\gamma$ decay. With a reasonable detector acceptance, a rejection of an order of magnitude could be achieved.

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FIG. 1. Angular dependence of partial decay width of $\mu^+ \to e^+\nu\overline{\nu}\gamma$ as a function of cosine of an angle between the photon and muon spin directions. Solid lines are the numerical calculations. Dotted lines are from Eq.(3). (a) $\delta y = 0.05$, (b) $\delta y = 0.03$, (c) $\delta y = 0.01$.

FIG. 2. Suppression factor η of accidental backgrounds as a function of a half of the opening angle of detection (θ_D) (a solid line for $P_\mu = 100$ % and a dotted line for $P_\mu = 97$ %).

Figure 1

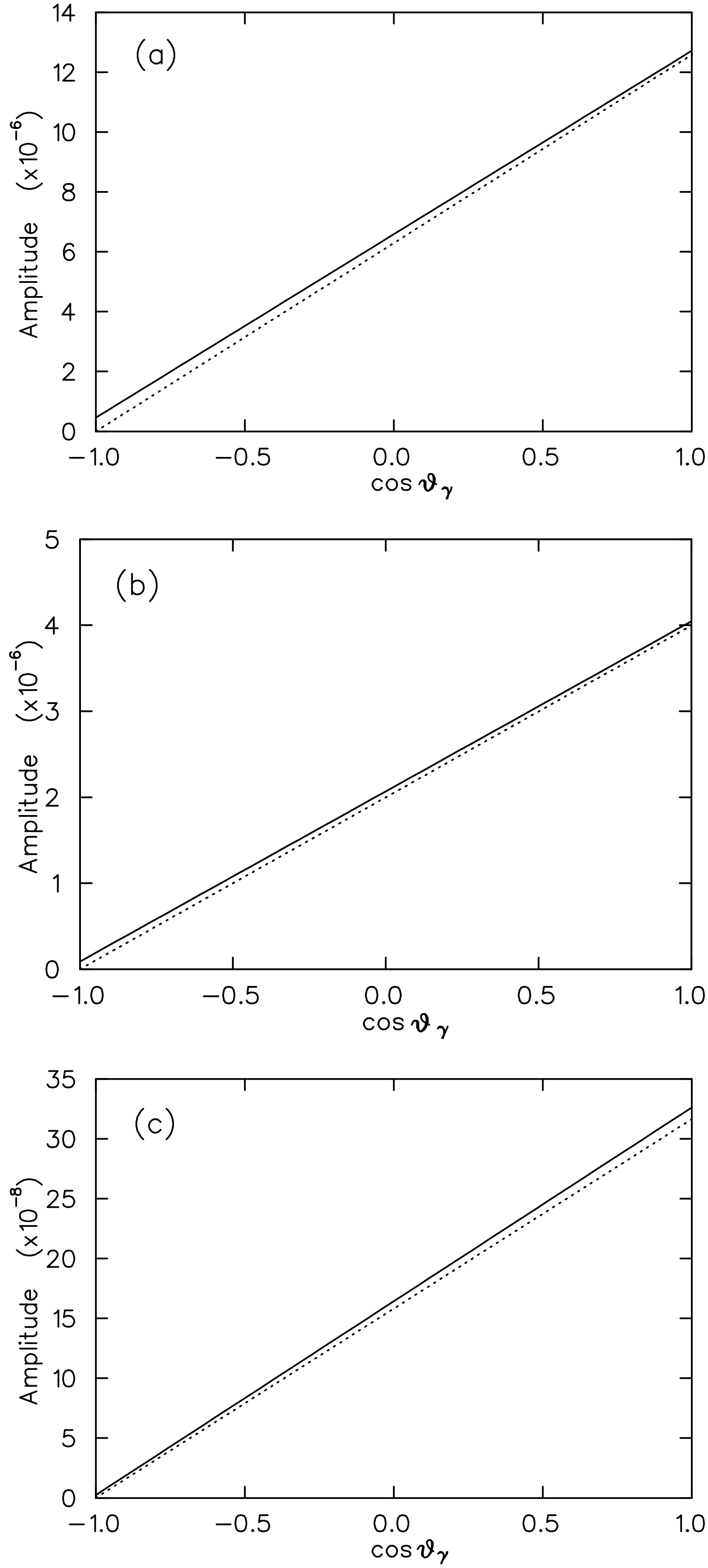
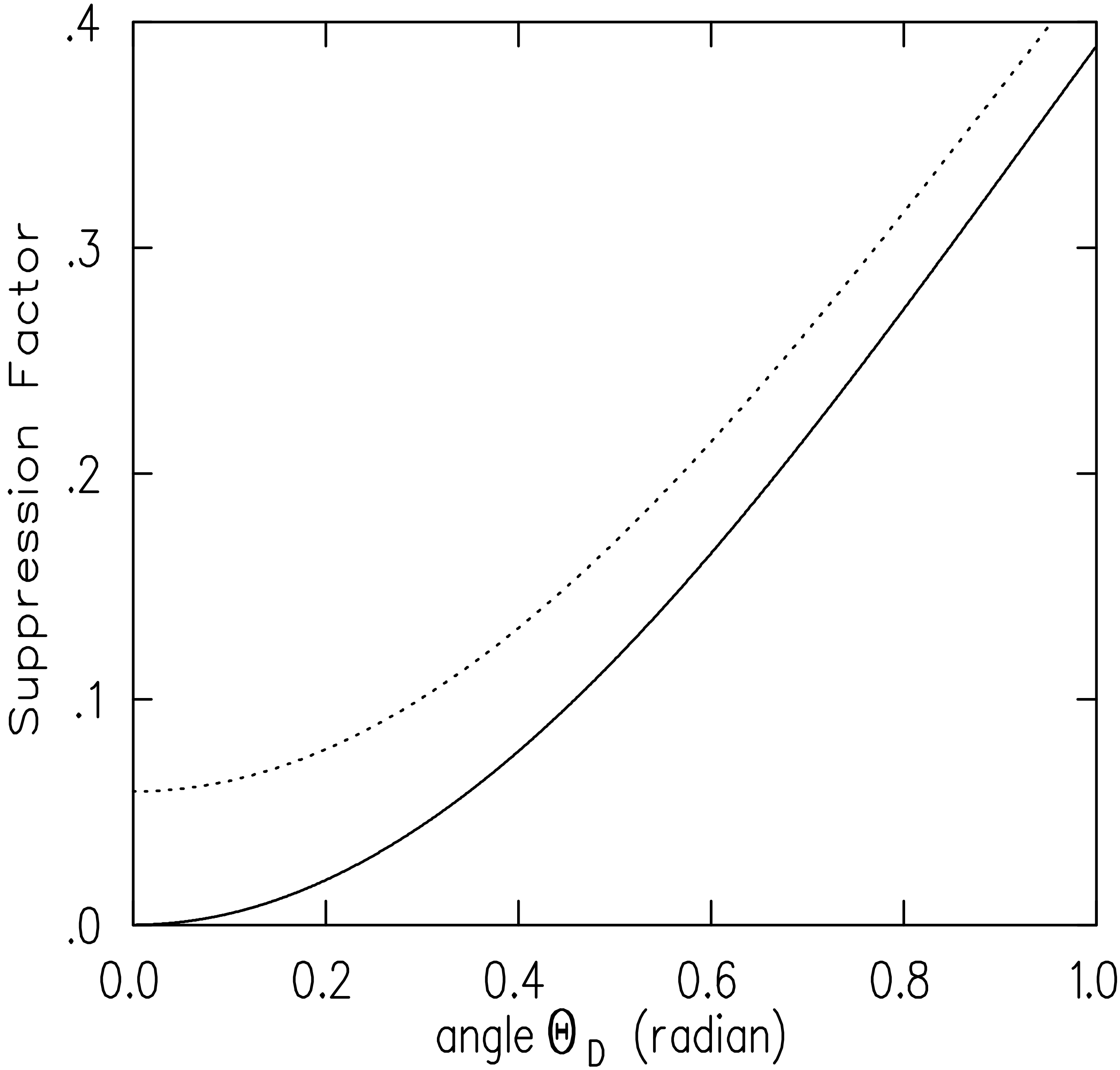

(a)
Amplitude (x10^{-6})
0
2
4
6
8
10
12
14
−1.0
−0.5
0.0
0.5
1.0
cos ϑ_γ
(b)
Amplitude (x10^{-6})
0
1
2
3
4
5
−1.0
−0.5
0.0
0.5
1.0
cos ϑ_γ
(c)
Amplitude (x10^{-8})
0
5
10
15
20
25
30
35
−1.0
−0.5
0.0
0.5
1.0
cos ϑ_γ

Figure 2



.4
.3
.2
.1
.0
Suppression Factor
0.0
0.2
0.4
0.6
0.8
1.0
angle Θ_D (radian)